Identification by data assimilation of the sensitivity of wavepackets in jets to non-linear effects

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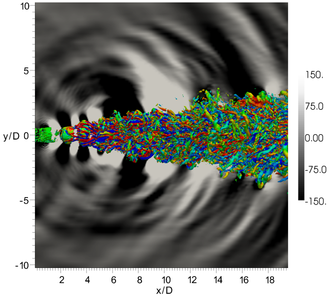
[6] California Institute of Technology, Pasadena, CA, USA.

Motivations

Context

Jet noise:

São Paulo airport.



G. Daviller (2011).

- Jet noise dominant during take off.
- Becomes limiting for specifications.
- Noise comes from the flow.

Motivations

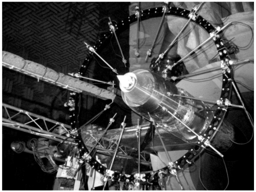
Wavepackets

Jordan & Colonius (2013)
Cavalieri et al. (2012,2013)
Tinney et al. (2008)

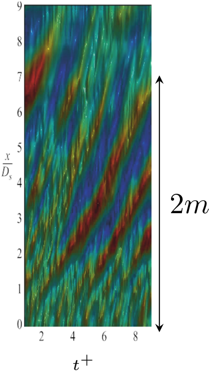
Where does the noise come from?

- Turbulent stochastic eddies?
- Or something more organised?

- Wavepackets in pressure/velocity field.



Tinney & Jordan 2008; Co-axial transonic heated jet $Re = 5 \times 10^6$.



Near-field pressure.

- Acoustic directivity as extended source *(low azimuthal angles)*.

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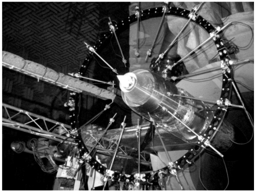

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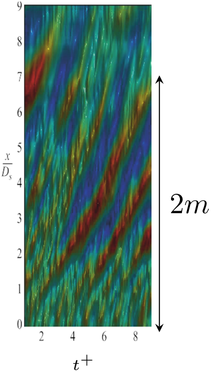

Near-field pressure.

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Source: likely a wavepacket shape.

Motivations

Jordan & Colonius (2013)
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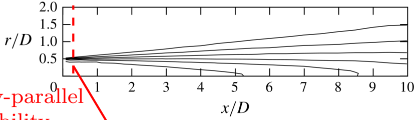
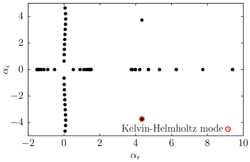
Sinha et al. (2014)
Baqui et al. (2015)

Wavepackets: Propagated linear instability waves

Experimental jet mean flow.



Locally-parallel
Stability



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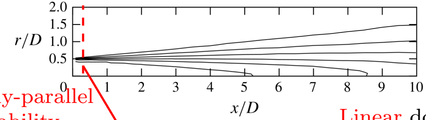

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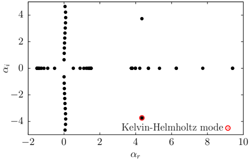
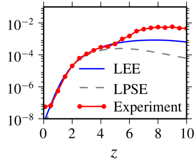


Linear downstream propagation of the Kelvin-Helmholtz mode.

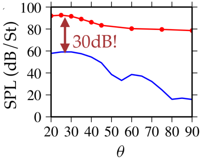
PSE

(*Parabolised Stability Eq.*)

Centerline $|u|^2$:



Far-field sound:



Non-linearities important for far-field prediction...

...toward low-order non-linear wavepacket model.

Motivations

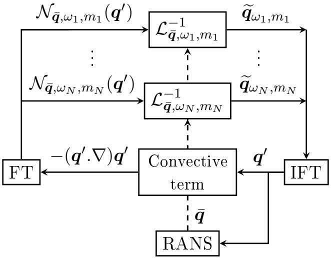
Non-linearity

Landhal (1967)
McKeon & Sharma (2010,2013)
Moarref et al. (2013)
Towne et al. (2015)

Non-linearity as an "external forcing"

Navier-Stokes in the frequency-azimuthal mode domain:

$$\mathcal{L}_{\bar{\boldsymbol{q}},\omega,m}\widetilde{\boldsymbol{q}}_{\omega,m} = \mathcal{N}_{\bar{\boldsymbol{q}},\omega,m}(\boldsymbol{q}').$$



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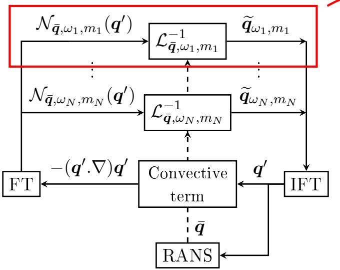

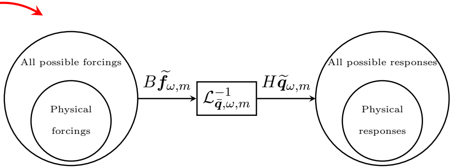

Identify relevant non-linearities.

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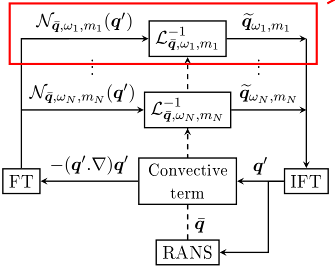

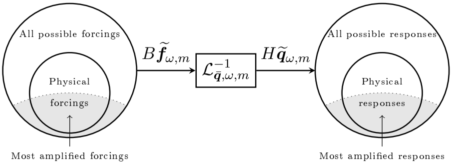

- Resolvent analysis.

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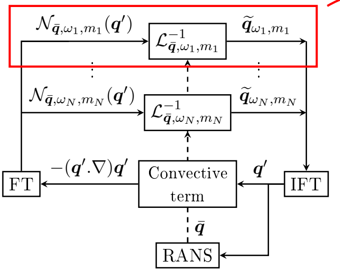

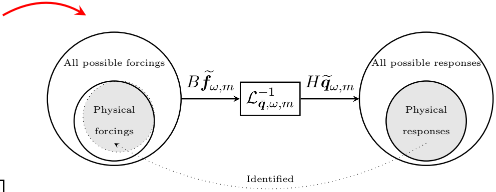

- Resolvent analysis.
- Inverse problem: PSE-4D-Var.

Identify relevant non-linearities.

Locally parallel resolvent analysis

Compressible Navier-Stokes equations:

$$
\begin{cases}
\dfrac{\partial \rho}{\partial t} + \nabla.(\rho \boldsymbol{u}) = 0 \\
\dfrac{\partial \rho \boldsymbol{u}}{\partial t} + (\boldsymbol{u}.\nabla)(\rho \boldsymbol{u}) = -\nabla p + \nabla.\tau \\
\rho \dfrac{\partial \gamma r T}{\partial t} + \rho(\boldsymbol{u}.\nabla)(\gamma r T) = -p(\nabla.\boldsymbol{u}) + \tau : \nabla \boldsymbol{u} + \dfrac{1}{RePr}\nabla.(\mu \nabla T)
\end{cases}
$$

with $\tau = \frac{1}{Re_a}\left(\mu\left(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right) + \left(\mu_B - \frac{2}{3}\mu\right)(\nabla.\boldsymbol{u})\mathbb{I}\right)$; $p = \rho r T$.

Locally parallel resolvent analysis

Model

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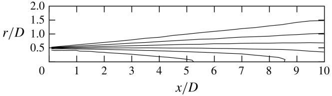
Linearised Navier-Stokes Equations: *over mean flow $U(y)$*

$$\left(\rho, \boldsymbol{u}, T\right)^T = \boldsymbol{q}_\phi(\boldsymbol{x}, t) = \bar{\boldsymbol{q}}(\boldsymbol{x}) + \boldsymbol{q}'(\boldsymbol{x}, t)$$



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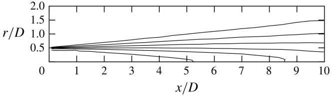

Locally parallel assumption + Fourier Transform:

$$\boldsymbol{q}'(x, r, \theta, t) = \frac{1}{(2\pi)^3} \sum_m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \widetilde{\boldsymbol{q}}_{\omega,m,\alpha}(r)\, e^{i(m\theta - \omega t + \alpha x)}\, \mathrm{d}\alpha\, \mathrm{d}\omega.$$

Locally parallel resolvent analysis

Model

Linearised problem: *eigenvalue problem* ($D(\alpha, \omega, m) = 0$).

$$\mathcal{L}_{\bar{\boldsymbol{q}},\alpha,\omega,m}\widetilde{\boldsymbol{q}}_{\alpha,\omega,m} = 0.$$

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Non-linear problem:

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(B restricts on $\boldsymbol{u}$ comp.)

Linearised problem with external forcing: *($D(\alpha,\omega,m)\neq 0$).*

$$H\widetilde{\boldsymbol{q}}_{\alpha,\omega,m}=H\mathcal{L}^{-1}_{\bar{\boldsymbol{q}},\alpha,\omega,m}B\widetilde{\boldsymbol{f}}_{\bar{\boldsymbol{q}},\alpha,\omega,m}.$$

(H restricts on $\boldsymbol{u}$ comp.)

Locally parallel resolvent analysis

Linearised problem with external forcing:

$$H\widetilde{\boldsymbol{q}}_{\alpha,\omega,m} = \underbrace{H\mathcal{L}^{-1}_{\bar{\boldsymbol{q}},\alpha,\omega,m}B}_{\text{SVD}}\,\widetilde{\boldsymbol{f}}_{\bar{\boldsymbol{q}},\alpha,\omega,m}.$$

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SVD to maximise Rayleigh quotient:

$$\max_{\widetilde{\boldsymbol{f}}_{\alpha,\omega,m}} \frac{\|H\widetilde{\boldsymbol{q}}_{\alpha,\omega,m}\|^2}{\|\widetilde{\boldsymbol{f}}_{\alpha,\omega,m}\|^2} = \frac{\|\left(H\mathcal{L}^{-1}_{\bar{\boldsymbol{q}},\alpha,\omega,m}B\right)\widetilde{\boldsymbol{f}}_{\alpha,\omega,m}\|^2}{\|\widetilde{\boldsymbol{f}}_{\alpha,\omega,m}\|^2}.$$

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Most amplified harmonic forcing/response modes:

$$H\mathcal{L}^{-1}_{\bar{\boldsymbol{q}},\alpha,\omega,m}B = U\Sigma V^*$$

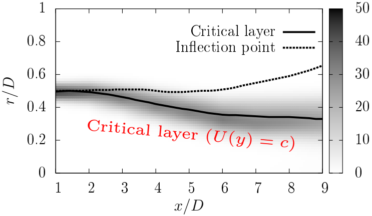
$$H\mathcal{L}^{-1}_{\bar{\boldsymbol{q}},\alpha,\omega,m}B\boldsymbol{V}_i = \sigma_i\boldsymbol{U}_i$$

with $U = (\boldsymbol{U}_1,\dots,\boldsymbol{U}_N)$, $V = (\boldsymbol{V}_1,\dots,\boldsymbol{V}_N)$ and $\Sigma = \mathrm{diag}(\sigma_1,\dots,\sigma_N)$.

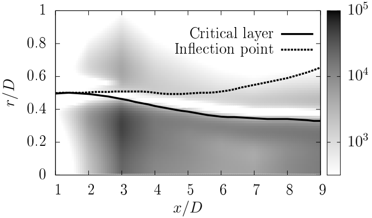
Locally parallel resolvent analysis

$m = 0$ (most acoustically efficient), St $= 0.6$, $\alpha = \alpha_{\mathrm{PSE}}$.

Optimal forcing $|u|^2$:



Optimal response $|u|^2$:



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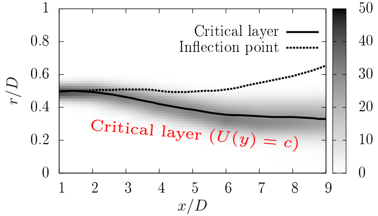

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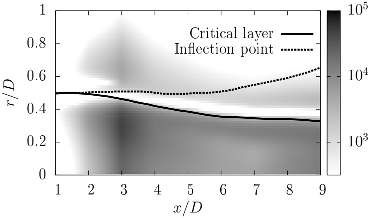
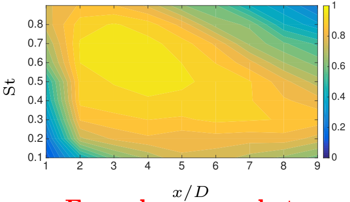
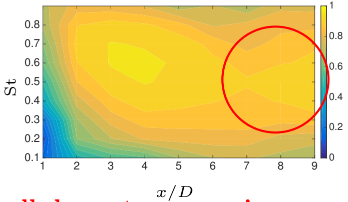


Normalised radial inner product

PSE vs Exp.:



Optimal response vs Exp.:



Forced wavepacket represents well downstream region.

PSE-4D-Var

Model

Parabolised stability equations:

Herbert (1997)

- ~~Locally parallel~~ → Slowly divergent.

$$\widetilde{\boldsymbol{q}}_{\omega,m}(x,r) = \boldsymbol{q}(x,r)\, e^{i\int_0^x \alpha(\xi)\,\mathrm{d}\xi}.$$

- We neglect viscosity.

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Model propagated downstream:

$$\begin{cases} E\dfrac{\partial \boldsymbol{q}}{\partial x} + (A + \alpha B)\boldsymbol{q} = 0 \\ \left(\boldsymbol{q}, \dfrac{\partial \boldsymbol{q}}{\partial x}\right)_r = 0 \\ \boldsymbol{q}(0) = \boldsymbol{q}_{\text{K-H}}, \end{cases}$$

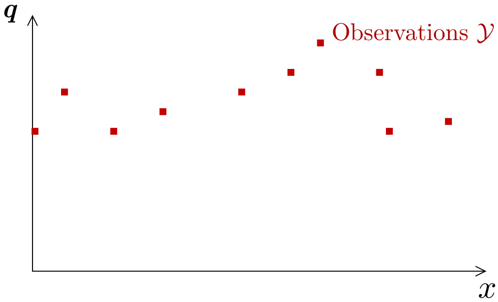
Inflow condition: Kelvin-Helmholtz mode from locally parallel stability analysis.

PSE-4D-Var

4D-Var

Observations: $\mathcal{Y}$: PSD $(|u|^2, |v|^2)^T$

Papadakis (2007)
Ansaldi (2015)



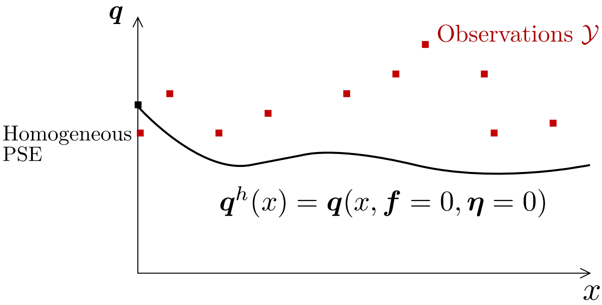
PSE-4D-Var

4D-Var

Model: Parabolised Stability Equations (PSE)

Papadakis (2007)
Ansaldi (2015)

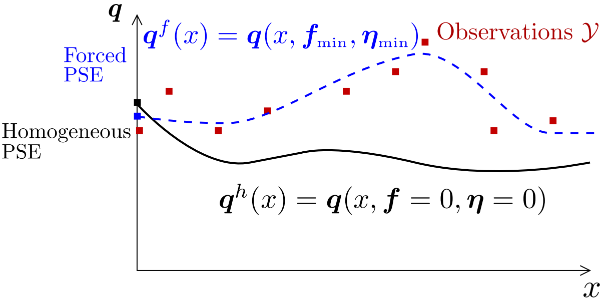
$$\begin{cases} E\dfrac{\partial \boldsymbol{q}}{\partial x} + (A+\alpha B)\boldsymbol{q} = \boldsymbol{f} \\ \left(\boldsymbol{q}, \dfrac{\partial \boldsymbol{q}}{\partial x}\right)_r = 0 \\ \boldsymbol{q}(0,r) = \boldsymbol{q}_0 + \boldsymbol{\eta}, \quad \alpha(0) = \alpha_0, \end{cases} \quad \text{with} \quad \widetilde{\boldsymbol{q}}_{\omega,m}(x) = \boldsymbol{q}(x) e^{i\int_0^{\xi} \alpha(\xi)\,\mathrm{d}\xi}$$



PSE-4D-Var

4D-Var: search for $(\boldsymbol{f}_{\min}, \boldsymbol{\eta}_{\min}) = \operatorname{argmin}(\mathcal{J}(\boldsymbol{q}, \alpha, \boldsymbol{f}, \boldsymbol{\eta}))$ *Papadakis (2007) Ansaldi (2015)*

$$\mathcal{J} = \frac{1}{2}\int_0^L \|\mathbb{H}(\boldsymbol{q}, \alpha) - \mathcal{Y}\|^2_{W_o}\, \mathrm{d}x + \frac{1}{2}\|\mathbb{H}_L - \mathcal{Y}_L\|^2_{W_T} + \frac{1}{2}\int_0^L \|\boldsymbol{f}\|^2_{W_f}\, \mathrm{d}x. + \frac{1}{2}\|\boldsymbol{\eta}\|^2_{W_\eta}.$$



PSE-4D-Var

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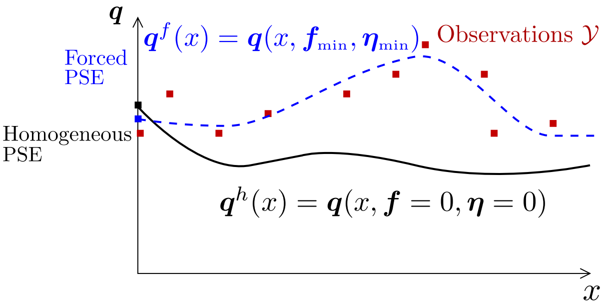
What are the missing non-linearities f in the linear model?

Solved using adjoint method (adjoint PSE).



PSE-4D-Var

4D-Var

Adjoint equation:

$$\begin{cases} -E^{+}\dfrac{\partial \boldsymbol{\lambda}}{\partial x} + \left(A + \alpha B - \dfrac{\partial E}{\partial x}\right)^{+} \boldsymbol{\lambda} + \dfrac{\partial \boldsymbol{q}}{\partial x}(\zeta - \zeta^{*}) - \boldsymbol{q}\dfrac{\partial \zeta^{*}}{\partial x} = RHS_1, \\ (B\boldsymbol{q}, \boldsymbol{\lambda})_r = RHS_2, \\ E^{+}\boldsymbol{\lambda}(L, r) = RHS_3, \\ \zeta(L) = 0. \end{cases}$$

Optimality condition:

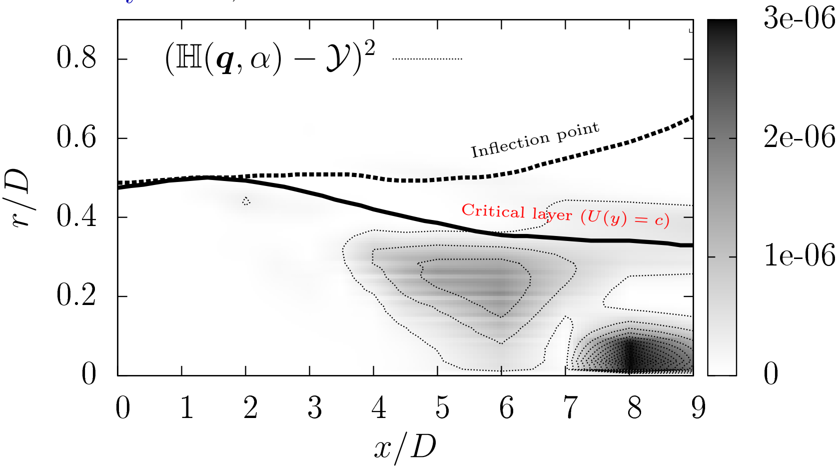
$$\begin{cases} \dfrac{\partial J}{\partial \boldsymbol{f}} = \boldsymbol{\lambda}(x, r) + W_f \boldsymbol{f} \\ \dfrac{\partial J}{\partial \boldsymbol{\eta}} = E^{+}\boldsymbol{\lambda}(0, r) + \boldsymbol{q}_0 \zeta^{*}(0) + W_\eta \boldsymbol{\eta}. \end{cases}$$

- Solved iteratively *(steepest descent)*.
- Weights determined by L-curve method.

PSE-4D-Var

Sensitivity

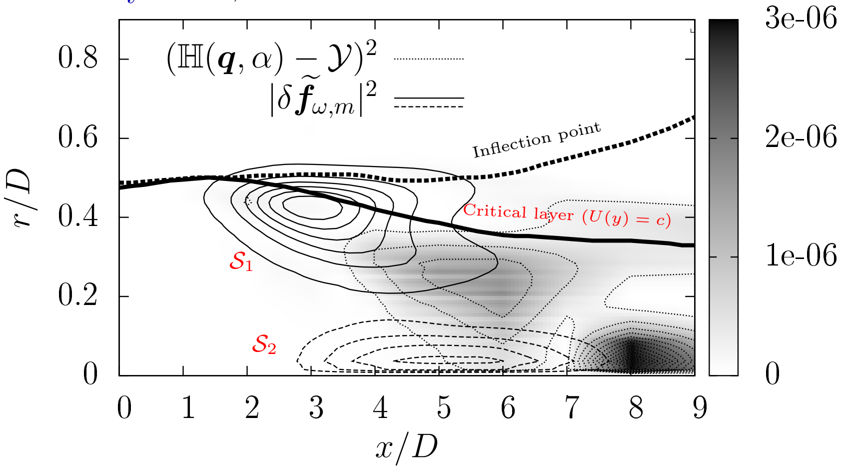
Sensitivity: $m = 0$, $\mathrm{St} = 0.6$.



$|u|^2$, grayscale=observation error.

PSE-4D-Var

Sensitivity: $m = 0$, $\mathrm{St} = 0.6$.



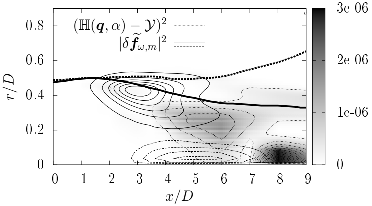
$|u|^2$, grayscale=observation error, contour=sensitivity.

PSE-4D-Var

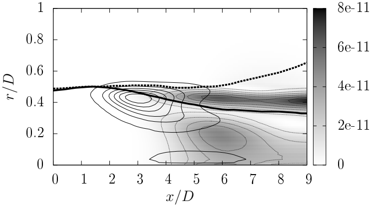
Sensitivity

Sensitivity: $m = 0$, $\mathrm{St} = 0.6$.

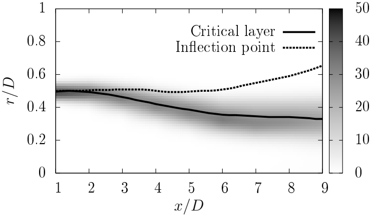
Error $|u|^2$:



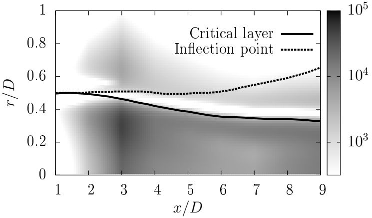
Response $|\delta u|^2 = |u^f - u^h|^2$:



Optimal forcing $|u|^2$:



Optimal response $|u|^2$:



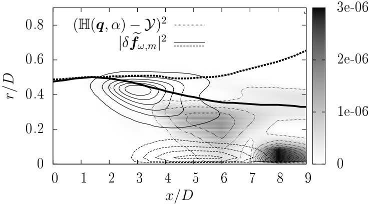
Predicted by locally parallel resolvent analysis.

PSE-4D-Var

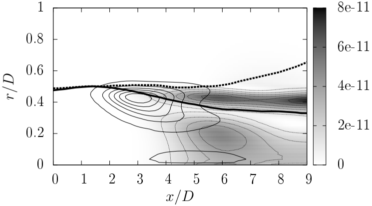
Sensitivity

Sensitivity: $m = 0$, $\mathrm{St} = 0.6$.

Error $|u|^2$:



Response $|\delta u|^2 = |u^f - u^h|^2$:



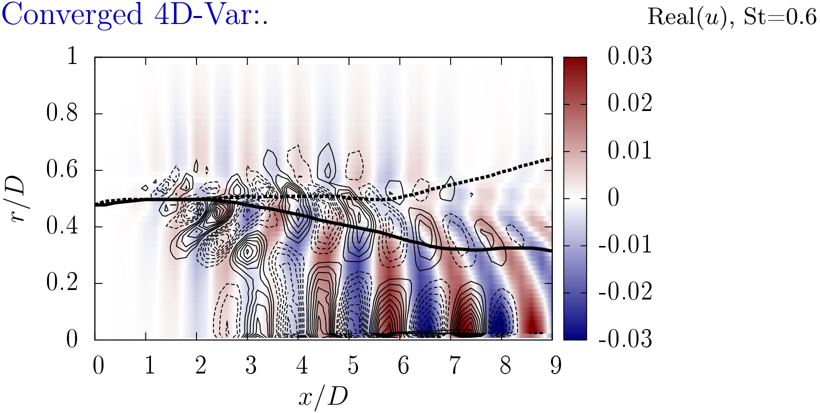
Inflow condition not sensitive:

- Critical layer sensitivity when neutral ($x \approx 3$).
- Kelvin-Helmholtz growth dominates upstream (modal behaviour).
- Homogeneous PSE works upstream.

PSE-4D-Var

Converged

Converged 4D-Var:.



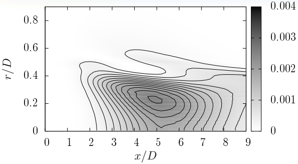
High forcing near:

- Critical layer.
- Centerline.

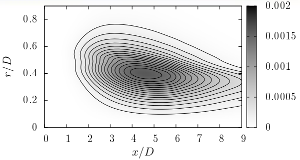
Converged results conserve the same trend.

PSE-4D-Var

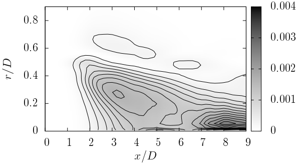
Converged



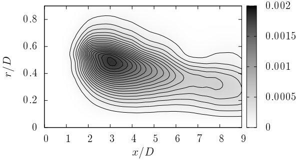
Homogeneous PSE $|\widetilde{\boldsymbol{q}}^u_{\omega,m}|^2$



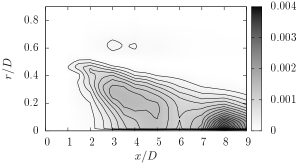
Homogeneous PSE $|\widetilde{\boldsymbol{q}}^v_{\omega,m}|^2$



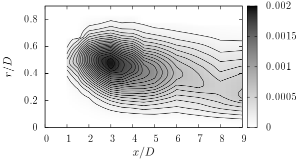
Forced PSE $|\widetilde{\boldsymbol{q}}^u_{\omega,m}|^2$



Forced PSE $|\widetilde{\boldsymbol{q}}^v_{\omega,m}|^2$



Observation $\mathcal{Y}^u$ **Match experiments.**



Observation $\mathcal{Y}^v$

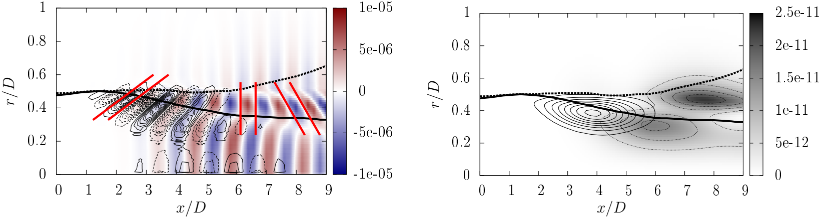
PSE-4D-Var

Orr mechanism

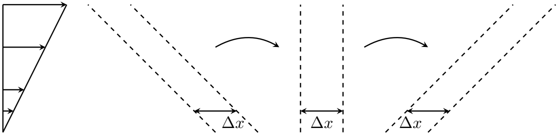
Orr (1907)
Boyd (1983)
Garnaud (2013)
Jiménez (2013,2015)

Sensitivity: contours=forcing; color=infinitesimal response.

Real(u): $|v|^2$:



Orr mechanism:



Vorticity conservation
$\Rightarrow$ Growth of $|v|^2$.

Tilting suggesting Orr mechanism in space.

PSE-4D-Var

Orr mechanism

Orr (1907)
Case (1960)

Orr model: for Couette flow ($U(y) = Sy$).

Temporal Orr model: *vorticity convected by shear*

$$\left(\frac{\partial \cdot}{\partial t} - Sy\frac{\partial \cdot}{\partial x}\right)\nabla^2\psi(x,y,t) = 0, \quad \psi \text{ stream func.}$$

$$\nabla^2\psi(x,y,t) = F(x - Syt, y).$$

Spatial Orr model: *Fourier Transform in time*

$$\nabla^2\widetilde{\psi}(x,y,\omega) = \widetilde{F}_2(y)\, e^{i\frac{\omega x}{Sy}}.$$

with

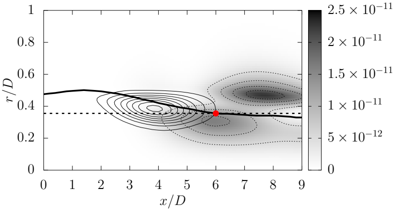
$$\widetilde{F}_2(y) = \frac{1}{Sy}\widetilde{F}\left(\frac{\omega}{Sy}, y\right) \quad \text{FFT in } x \text{ of } F(x,y).$$

$$\boxed{\widetilde{F}_2(y) = \nabla^2\widetilde{\psi}(0,y,\omega)}$$

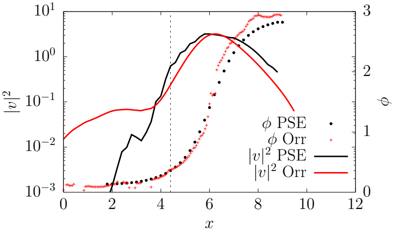
PSE-4D-Var

Orr mechanism

Jet locally approximated by Couette flow!



Point used for matching flows (max(v) at critical layer).



Comparison PSE sensitivity / Orr model.

Orr mechanism quantitatively confirmed.

Conclusion

Summary

- Forced wavepacket consistent with experiment.
- Critical layer highly sensitive region to non-linearity.
- Along the critical layer, shear convects and tilts the response to non-linearities, leading to an amplification by Orr mechanism.
- PSE-4D-Var powerful tool.